Where is the Science in the Software Sciences?

MAPi 2017

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Informatics ≃ Computer "Science"?

Vinton Cerf (1943-)

ACM President

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Where is the Science in Computer Science?

"... we have a responsibility to pursue the science in computer science. We must develop better tools and much deeper understanding of the systems we invent and a far greater ability to make predictions about the behavior of these complex, connected, and interacting systems.".

(Vinton G. Cerf, Letter from the ACM President, CACM 55(10), Oct. 2012)

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for $i = 1$ *to* 4 **do** $\{\, industry\ (i); \,\}$

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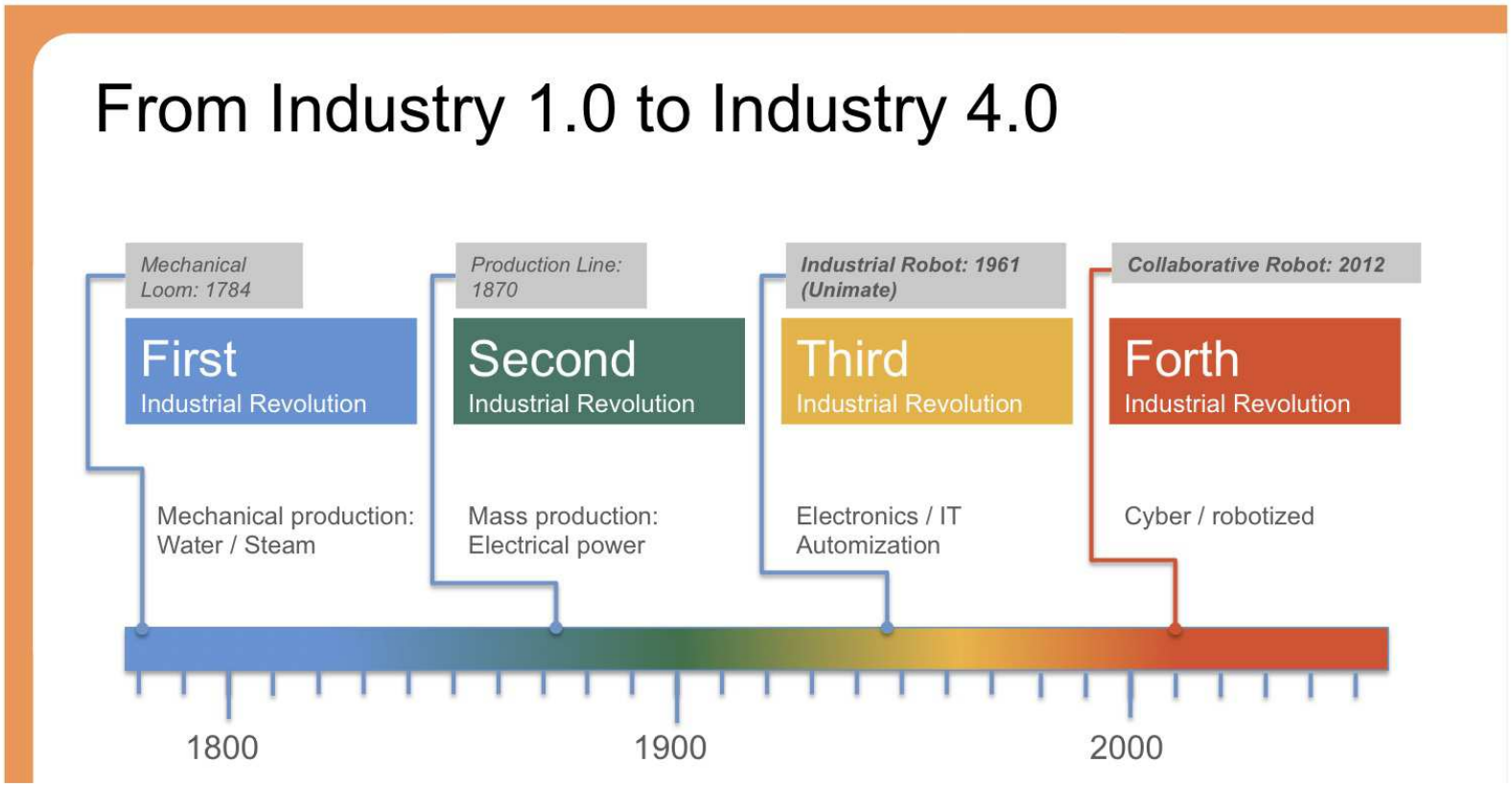
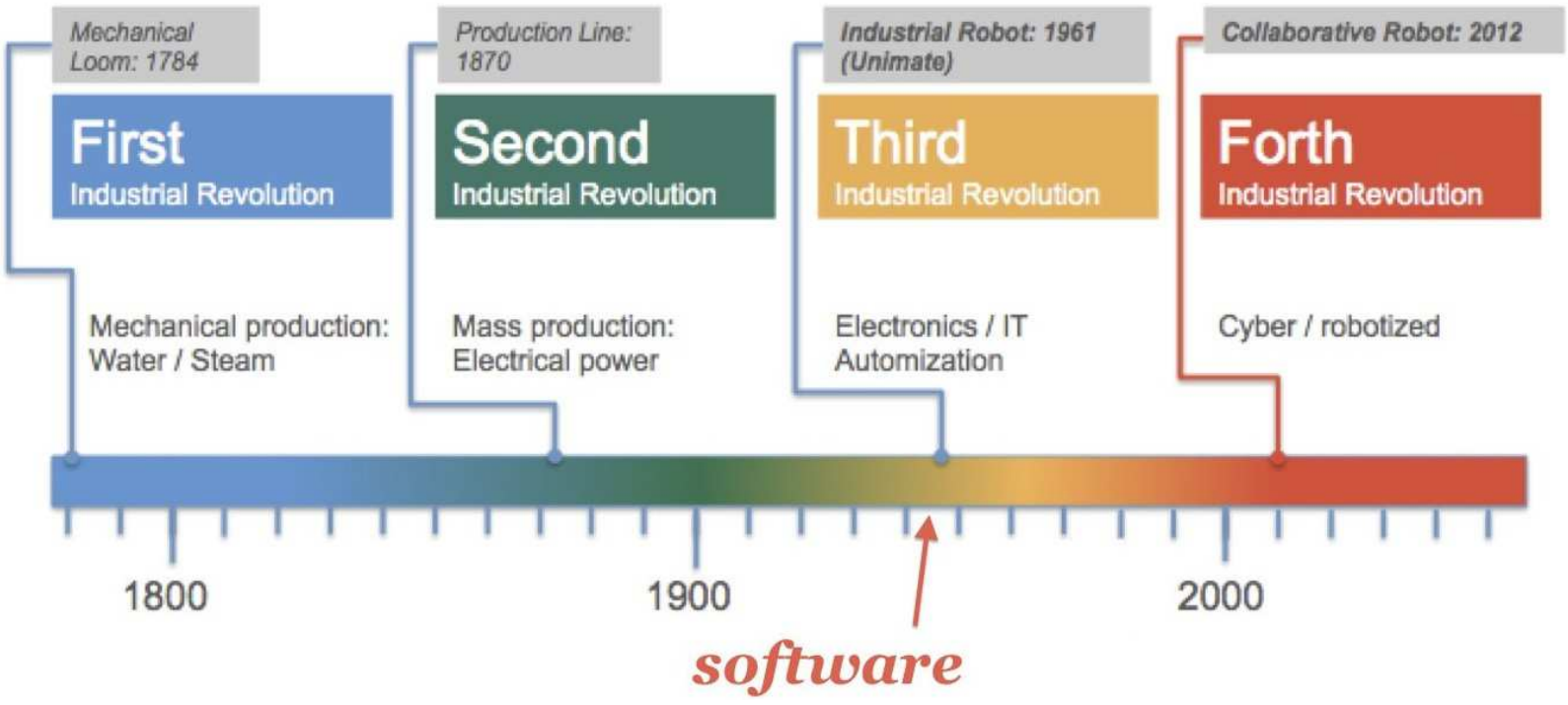


Credit: nelmia Robotics Insight (2015)

Turning point



("L'enfant terrible" is born)

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Crisis

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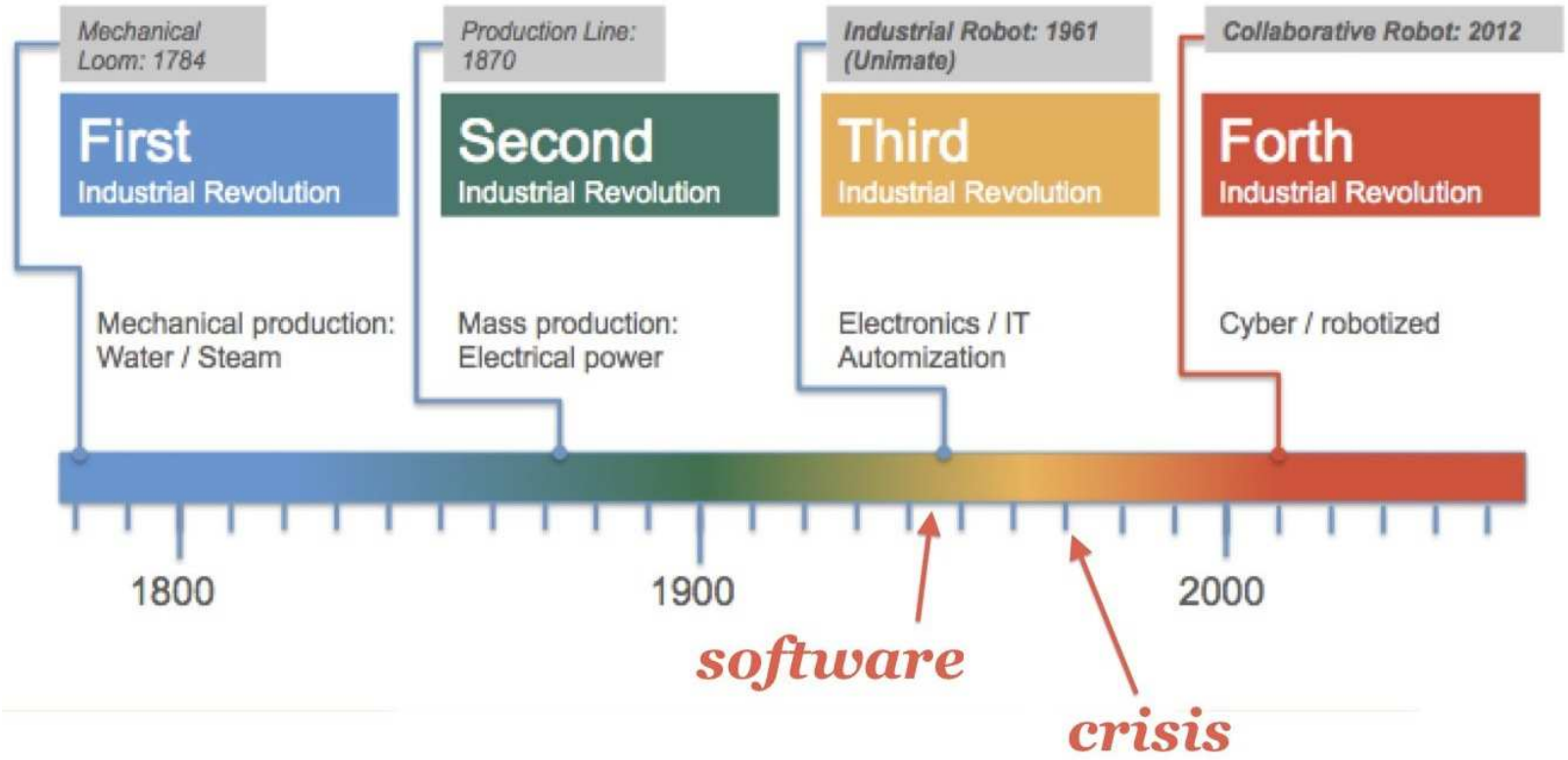

1st NATO Conference on Software Engineering, Darmstadt, October 1968

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Meanwhile (50 years)

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Software {
- **Process** — 
- **Product** — 

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"L'enfant terrible"

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Hardware and other "traditional" industrial **products** fabricated according to the laws of **physics**.

Software is not governed by the laws of physics:

- it does not weight / does not smell
- it does not warm up / cool down
- it is chemically neutral ...

Anthony Oettinger (ACM President, 1967):

"(...) the scientific, rigorous component of computing, is more like **mathematics** *than it is like* **physics".**

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Software = mathematics in motion

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Can one *pretend* that **software production** is not affected by its **special nature** and move on?

Many people have tried to do so for 50 years...

Still Oettinger **(already** in 1967):

"It is a matter of **complexity**. *Once you start putting thousands of these instructions together you create a* **monster** *which is* **unintelligible** *to anyone save its creator and, most of the time, unfortunately even to the creator."*

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1976 — vintage year

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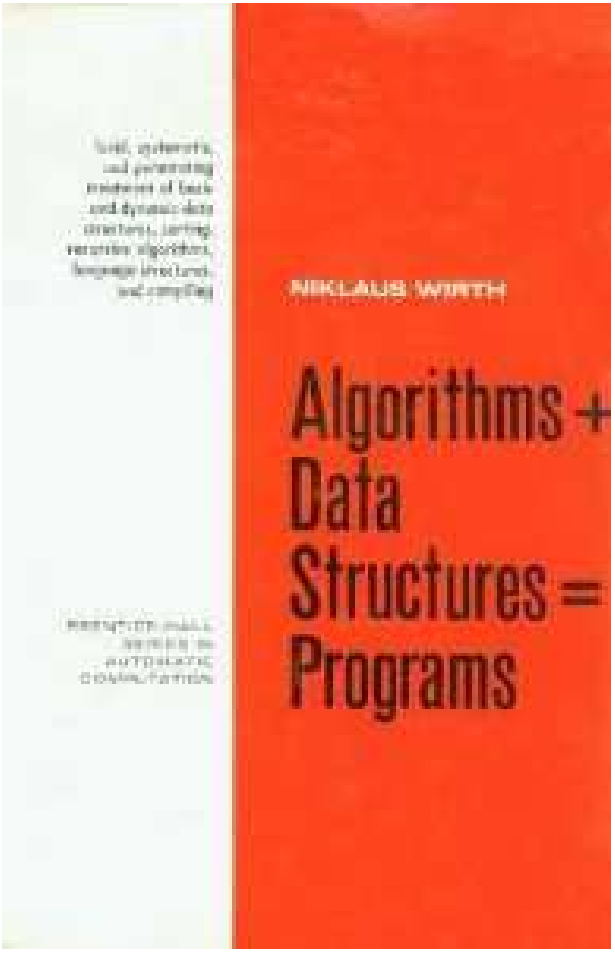
Famous textbook by Niklaus Wirth (1934-) that inspired **structured programming** and made history in its time.

Published in 1976.

PASCAL — the language "par exellence" (mostly in Europe).

No **Java** yet. Object orientation stimulated by MODULA, a PASCAL derivative.



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1976 — vintage year

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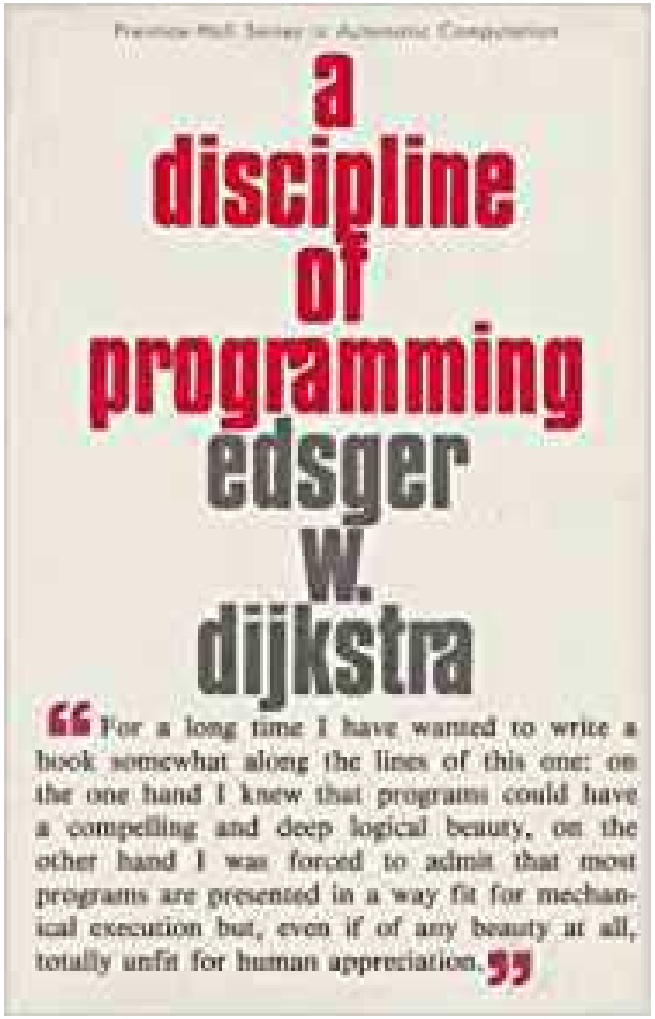
The "textbook of all textbooks".

Emphasis on elegance and abstraction.

Also published in 1976.

Programming as a scientific activity.

E.W. Dijkstra (1930-2002), Turing Award, one of the most eminent computer scientists ever.



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Shortly afterwards

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Famous "dragon book" by A.V. Aho (1941-) e J.D. Ullman (1942-).

Published in 1977.

Inspiration for generations of compiler writers.

Compiler writing made systematic.

Scientific basis: *The Theory of Parsing, Translating & Compiling* (1972) by the same authors.

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Still people complain (1976)

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E.W. Dijkstra (1920-2002)

(...) the term "Database Technology", although sometimes used, is immature, for there is hardly any underlying "science" that could justify the use of the term "technology".

(...) They seem to form an inbred crowd with very little knowledge of computing science in general (...) Often they seemed to be mentally trapped by the intricacies of early, rather **ad hoc** *solutions (...)*

(E.W. Dijkstra, EWD577, 1976)

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Reaction (early 1980s)

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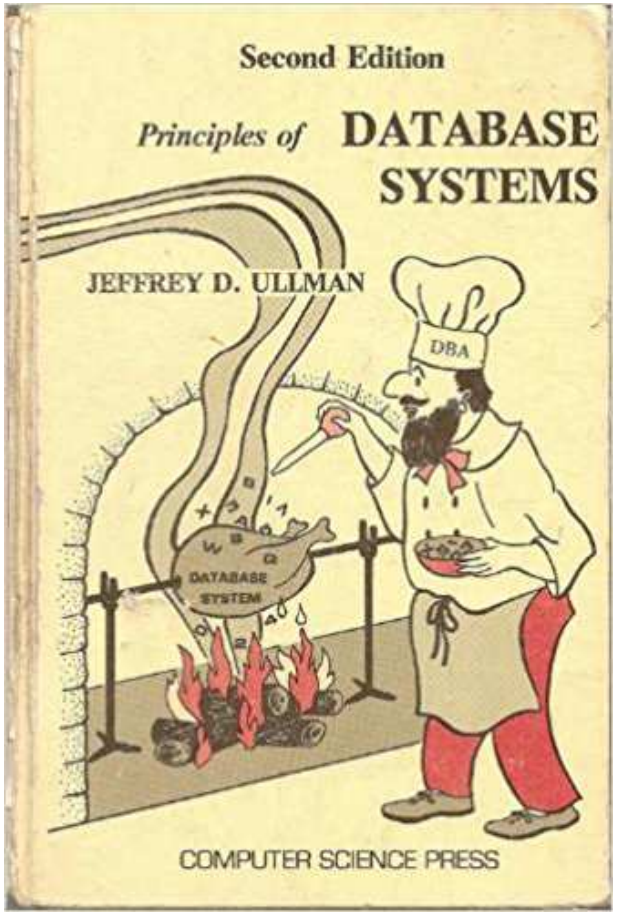

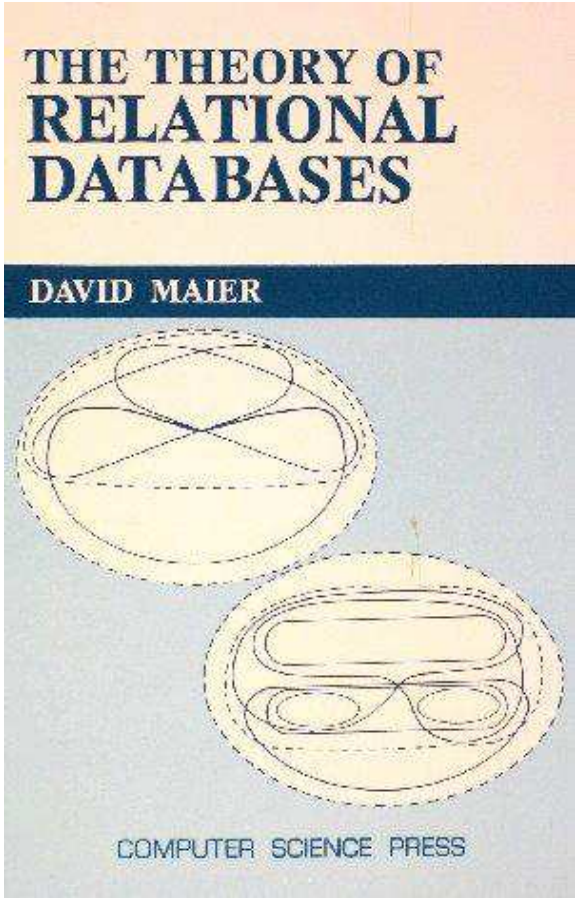

Relational database science — books by Ullman (1982) and Maier (1983).

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>30 years later

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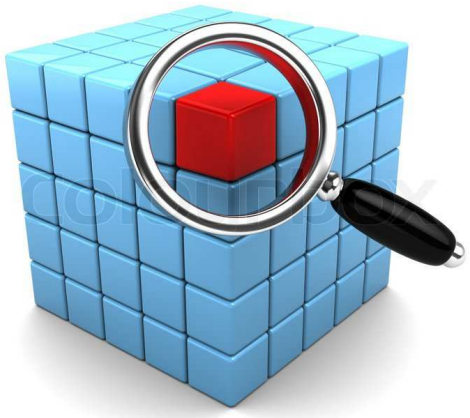

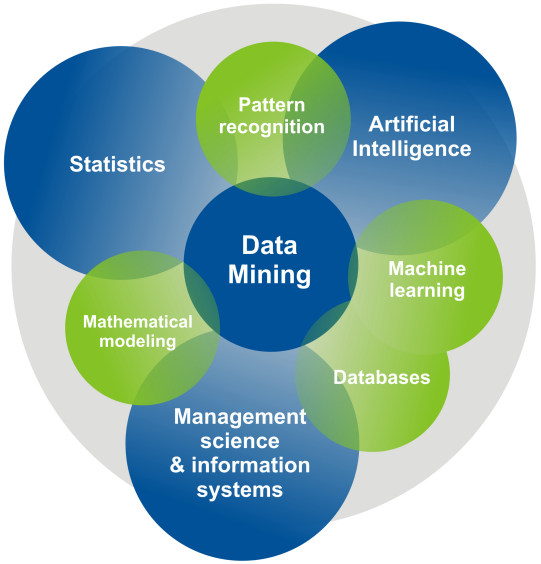

Have scientific standards kept up with so higher demands in technology?

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A much older concern

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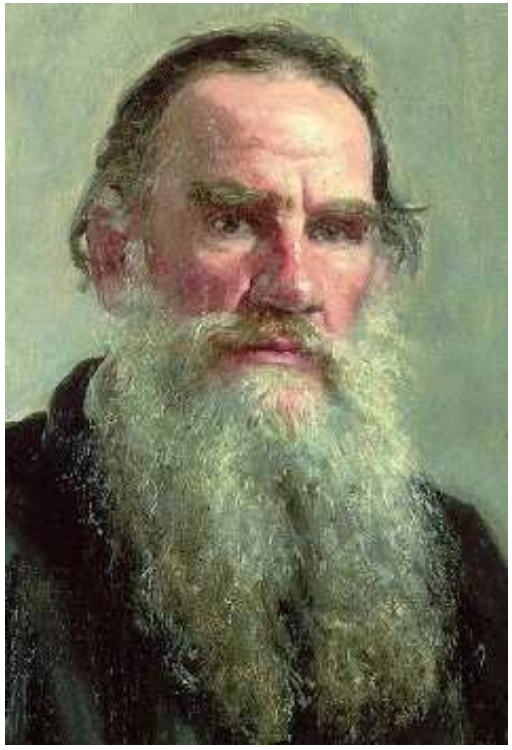
"Only by taking infinitesimally small units for observation (the **differential** *of history, that is, the individual tendencies of men) and attaining to the art of* **integrating** *them (that is, finding the sum of these infinitesimals) can we hope to arrive at the laws of history."*

Leo Tolstoy, "War and Peace"
- Book XI, Chap.II (1869)



L. Tolstoy (1828–1910)

The oldest definition I know of **data-mining**.

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The age of data

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Note the **maths** flavour of Tolstoy's text

How maths-strong is data-mining?

Have we attained the "**art of integration**" foreseen by Tolstoy?

Again the **glorious 1970s** — since the early days of psychometrics in the **social sciences** — **linear algebra** (LA) central to data analysis (e.g. Tucker tensor decompositions etc).

All done? Anything else to do?

L.R. Tucker (1910-2004)

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Yes, perhaps...

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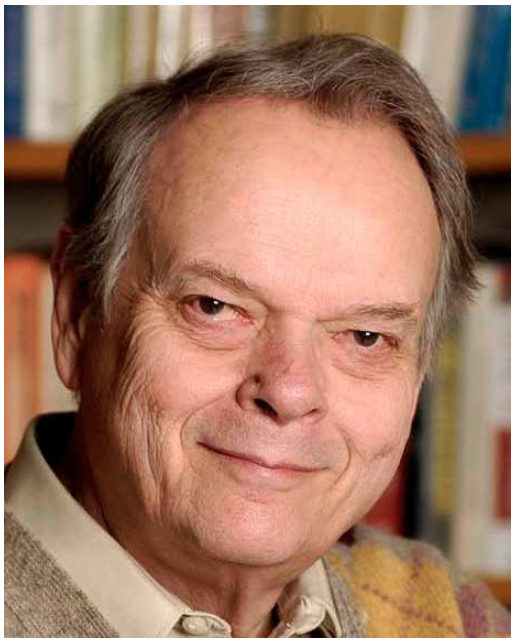
Data mining seems to be ignoring the role of **types** and **type parametricity** in software — one of the most significant advances in CS.

Nice theory called **parametric polymorphism** (John Reynolds, CMU).

So nice that you can derive **properties** of your (typed) operations **before writing them** (!)

What can **parametricity** possibly mean in the **data-analysis** setting?



J.C. Reynolds
(1935–2013)

Example — parametric data cubes

Back to square one — some raw data:

$t =$

#	Model	Year	Color	Sale
1	*Chevy*	1990	*Red*	5
2	*Chevy*	1990	*Blue*	87
3	*Ford*	1990	*Green*	64
4	*Ford*	1990	*Blue*	99
5	*Ford*	1991	*Red*	8
6	*Ford*	1991	*Blue*	7

Columns — attributes — the **observables**
Rows — records (n-many) — the **infinitesimals**

Column-orientation — each column (attribute) A represented by a function $t_A : n \to A$ such that $a = t_A\,(i)$ means "a is the value of attribute A in record nr i".

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Records are tuples

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Can records be rebuilt from such **attribute projection** functions?

Yes — by **tupling** them.

Tupling: *Given functions* $f : A \to B$ *and* $g : A \to C$, *their tupling is the function* $f \triangledown g$ *such that*

$$(f \triangledown g)\ a = (f\ a, g\ a)$$

For instance,

$$(t_{Color} \triangledown t_{Model})\ 2 = (Blue, Chevy),$$
$$(t_{Year} \triangledown (t_{Color} \triangledown t_{Model}))\ 3 = (1990, (Green, Ford))$$

and so on.

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The typed LA approach

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Represent functions by Boolean matrices.

Given (finite) types A and B, any function

$$f : A \to B$$

can be represented by a matrix $[\![f]\!]$ with A-many columns and B-many rows such that, for any $b \in B$ and $a \in A$, matrix cell

$$b\ [\![f]\!]\ a = \begin{cases} 1 \Leftarrow b = f\ a \\ 0\ \mathit{otherwise} \end{cases}$$

NB: Following the **infix** notation usually adopted for relations (which are Boolean matrices) — for instance $y \leqslant x$ — we write $y\ M\ x$ to denote the contents of the cell in matrix M addressed by row y and column x.

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The typed LA approach

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One projection function (matrix) per **dimension** attribute:

t_{Model}	1	2	3	4	5	6
Chevy	1	1	0	0	0	0
Ford	0	0	1	1	1	1

t_{Year}	1	2	3	4	5	6
1990	1	1	1	1	0	0
1991	0	0	0	0	1	1

t_{Color}	1	2	3	4	5	6
Blue	0	1	0	1	0	1
Green	0	0	1	0	0	0
Red	1	0	0	0	1	0

#	**Model**	**Year**	**Color**	**Sale**
1	Chevy	1990	Red	5
2	Chevy	1990	Blue	87
3	Ford	1990	Green	64
4	Ford	1990	Blue	99
5	Ford	1991	Red	8
6	Ford	1991	Blue	7

NB: we tend to abbreviate $[\![f]\!]$ by f where the context is clear.

The typed LA approach

Note how the inverse of a function is also represented by a Boolean matrix, e.g.

t°_{Model}	Chevy	Ford
1	1	0
2	1	0
3	0	1
4	0	1
5	0	1
6	0	1

versus

t_{Model}	1	2	3	4	5	6
Chevy	1	1	0	0	0	0
Ford	0	0	1	1	1	1

— no longer a function, but still a relation, a Boolean matrix :-)

Clearly,

$$j\ t^{\circ}_{Model}\ a = a\ t_{Model}\ j$$

Given a matrix M, M° is known as the **transposition** of M.

The typed LA approach

We **type** matrices in the same way as functions: $M : A \to B$ means a matrix M with A-many columns and B-many rows.

Matrices are **arrows**: $A \xrightarrow{M} B$ denotes a matrix from A (source) to B (target), where A, B are (finite) types.

Writing $B \xleftarrow{M} A$ means the same as $A \xrightarrow{M} B$.

Composition — *aka* matrix multiplication:

$$B \xleftarrow{M} A \xleftarrow{N} C \qquad M \cdot N : A \to C$$

$$b(M \cdot N)c \;=\; \langle \sum\, a \;::\; (b\; M\; a) \times (a\; N\; c) \rangle$$

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The typed LA approach

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Function composition implemented by matrix multiplication, $[\![f \cdot g]\!] = [\![f]\!] \cdot [\![g]\!]$

Identity — the identity matrix id corresponds to the identity function and is such that

$$M \cdot id = M = id \cdot M \tag{1}$$

Function tupling corresponds to the so-called **Khatri-Rao product** $M \triangledown N$ defined index-wise by

$$(b, c)\ (M \triangledown N)\ a \quad = \quad (b\ M\ a) \times (c\ N\ a) \tag{2}$$

Khatri-Rao is a "column-wise" version of the well-known **Kronecker product** $M \otimes N$:

$$(y, x)\ (M \otimes N)\ (b, a) \quad = \quad (y\ M\ b) \times (x\ N\ a) \tag{3}$$

Typing data

The raw data given above is represented in typed LA by the expression

$$v = (t_{Year} \triangledown (t_{Color} \triangledown t_{Model})) \cdot (t^{Sale})^{\circ}$$

of type

$$v : 1 \to (Year \times (Color \times Model))$$

depicted aside.

Year x (Color x Model)			ALL
1990	Blue	Chevy	87
		Ford	99
	Green	Chevy	0
		Ford	64
	Red	Chevy	5
		Ford	0
1991	Blue	Chevy	0
		Ford	7
	Green	Chevy	0
		Ford	0
	Red	Chevy	0
		Ford	8

v is a **multi-dimensional** column vector — a **tensor**. Datatype $1 = \{\text{ALL}\}$ is the so-called **singleton** type.

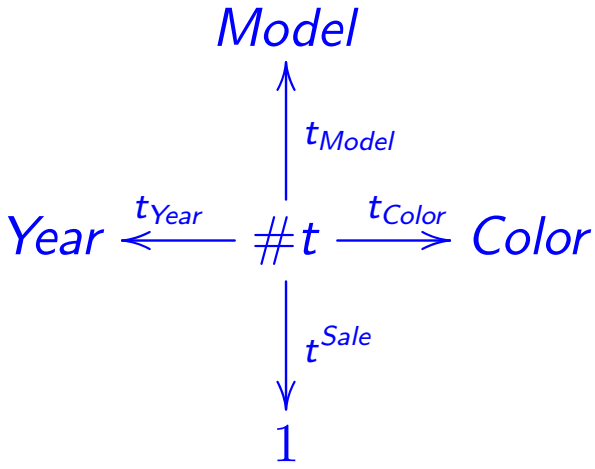
Dimensions and measures

Sale is a special kind of data — a **measure**. Measures are encoded as **row** vectors, e.g.

t^{Sale}	1	2	3	4	5	6
1	5	87	64	99	8	7

recall

#	Model	Year	Color	Sale
1	Chevy	1990	Red	5
2	Chevy	1990	Blue	87
3	Ford	1990	Green	64
4	Ford	1990	Blue	99
5	Ford	1991	Red	8
6	Ford	1991	Blue	7



Summary: **dimensions** *are* **matrices**, **measures** *are* **vectors**.

Measures provide for **integration** in Tolstoy's sense — *aka* **consolidation**

Totalisers

There is a unique function in type $A \to 1$, usually named $A \xrightarrow{!} 1$. This corresponds to a row vector wholly filled with 1s.

Example: $2 \xrightarrow{!} 1 = \begin{bmatrix} 1 & 1 \end{bmatrix}$

Given $M : B \to A$, the expression $! \cdot M$ (where $A \xrightarrow{!} 1$) is the row vector (of type $B \to 1$) that contains all column **totals** of M,

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 50 & 40 & 85 & 115 \\ 50 & 10 & 85 & 75 \end{bmatrix} = \begin{bmatrix} 100 & 50 & 170 & 190 \end{bmatrix}$$

Given type A, define its **totalizer** matrix $A \xrightarrow{\tau_A} A + 1$ by

$$\begin{aligned} \tau_A &: A \to A + 1 \\ \tau_A &= \begin{bmatrix} id \\ \hline ! \end{bmatrix} \end{aligned} \tag{5}$$

Thus $\tau_A \cdot M$ yields a copy of M on top of the corresponding totals.

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Cubes

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Data **cubes** can be obtained from products of totalizers.

Recall the Kronecker (tensor) product $M \otimes N$ of two matrices $A \xrightarrow{M} B$ and $C \xrightarrow{N} D$, which is of type $A \times C \xrightarrow{M \otimes N} B \times D$.

The matrix

$$A \times B \xrightarrow{\tau_A \otimes \tau_B} (A+1) \times (B+1)$$

provides for totalization on the **two dimensions** A and B.

Indeed, type $(A+1) \times (B+1)$ is isomorphic to $A \times B + A + B + 1$, whose four parcels represent the four elements of the "**dimension powerset** of $\{A, B\}$".

Cube = muti-dimensional totalisation

Recalling

$$v = (t_{Year} \triangledown (t_{Color} \triangledown t_{Model})) \cdot (t^{Sale})^{\circ}$$

build

$$c = (\tau_{Year} \otimes (\tau_{Color} \otimes \tau_{Model})) \cdot v$$

This is the multidimensional vector (tensor) representing the **data cube** for

- **dimensions** *Year*, *Color*, *Model*
- **measure** *Sale*

depicted aside.

(Year+1) x ((Color+1) x (Model+1))			ALL
1990	Blue	Chevy	87
		Ford	99
		ALL	186
	Green	Chevy	0
		Ford	64
		ALL	64
	Red	Chevy	5
		Ford	0
		ALL	5
	ALL	Chevy	92
		Ford	163
		ALL	255
1991	Blue	Chevy	0
		Ford	7
		ALL	7
	Green	Chevy	0
		Ford	0
		ALL	0
	Red	Chevy	0
		Ford	8
		ALL	8
	ALL	Chevy	0
		Ford	15
		ALL	15
ALL	Blue	Chevy	87
		Ford	106
		ALL	193
	Green	Chevy	0
		Ford	64
		ALL	64
	Red	Chevy	5
		Ford	8
		ALL	13
	ALL	Chevy	92
		Ford	178
		ALL	270

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Generalizing data cubes

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In our approach a **cube** is not necessarily one such column vector.

The key to generic data cubes is (generic) **vectorization**, a kind of "**matrix currying**": given $A \times B \xrightarrow{M} C$ with $A \times B$-many columns and C-many rows, reshape M into its **vectorized** version $B \xrightarrow{\mathbf{vec}_A\, M} A \times C$ with B-many columns and $A \times C$-many rows.

Matrices M and $\mathbf{vec}_A\, M$ are **isomorphic** — they contain the same information in different formats, cf.

$$c\ M\ (a, b) \quad = \quad (a, c)\ (\mathbf{vec}_A\, M)\ b \tag{6}$$

for every a, b, c.

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Generalizing data cubes

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Vectorization thus has an inverse operation — **unvectorization**:

$$A \times B \to C \quad \underset{\mathbf{unvec}_A}{\overset{\mathbf{vec}_A}{\cong}} \quad B \to A \times C$$

That is, M can be retrieved back from $\mathbf{vec}_A\, M$ by unvectorizing it:

$$N = \mathbf{vec}_A\, M \quad \Leftrightarrow \quad \mathbf{unvec}_A\, N = M \tag{7}$$

Vectorization has a rich algebra, e.g. a **fusion**-law

$$(\mathbf{vec}\, M) \cdot N \quad = \quad \mathbf{vec}\, (M \cdot (id \otimes N)) \tag{8}$$

and an **absorption**-law:

$$\mathbf{vec}\, (M \cdot N) \quad = \quad (id \otimes M) \cdot \mathbf{vec}\, N \tag{9}$$

(De)vectorization

Devectorizing our starting tensor, across dimension *Year*:

$$\textit{Year} \times (\textit{Color} \times \textit{Model}) \xleftarrow{\quad} 1$$

$$\textbf{\textit{unvec}}_{\textit{Year}} \left(\begin{array}{lll|l} & & & \text{ALL} \\ \hline \multirow{6}{*}{1990} & \multirow{2}{*}{Blue} & Chevy & 87 \\ & & Ford & 99 \\ & \multirow{2}{*}{Green} & Chevy & 0 \\ & & Ford & 64 \\ & \multirow{2}{*}{Red} & Chevy & 5 \\ & & Ford & 0 \\ \hline \multirow{6}{*}{1991} & \multirow{2}{*}{Blue} & Chevy & 0 \\ & & Ford & 7 \\ & \multirow{2}{*}{Green} & Chevy & 0 \\ & & Ford & 0 \\ & \multirow{2}{*}{Red} & Chevy & 0 \\ & & Ford & 8 \end{array} \right) =$$

$$\textit{Color} \times \textit{Model} \xleftarrow{\quad} \textit{Year}$$

		1990	1991
Blue	Chevy	87	0
	Ford	99	7
Green	Chevy	0	0
	Ford	64	0
Red	Chevy	5	0
	Ford	0	8

There is room for further unvectorizing the outcome, this time across *Color* — next slide:

(De)vectorization

Further unvectorization:

$$\textbf{unvec}_{Color}\left(\begin{array}{|ll|cc|}\hline & & 1990 & 1991 \\ \hline Blue & Chevy & 87 & 0 \\ & Ford & 99 & 7 \\ \hline Green & Chevy & 0 & 0 \\ & Ford & 64 & 0 \\ \hline Red & Chevy & 5 & 0 \\ & Ford & 0 & 8 \\ \hline\end{array}\right)_{Color \times Model \longleftarrow Year} = \underset{Model \longleftarrow Color \times Year}{}$$

	Blue		Green		Red	
	1990	1991	1990	1991	1990	1991
Chevy	87	0	0	0	5	0
Ford	99	7	64	0	0	8

and so on.

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Generic cubes

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It turns out **that** cubes can be calculated for any such two-dimensional versions of our original data tensor, for instance,

$$\textbf{cube}\ N : \ \mathit{Model} + 1 \longleftarrow (\mathit{Color} + 1) \times (\mathit{Year} + 1)$$

$$\textbf{cube}\ N = \tau_{\mathit{Model}} \cdot N \cdot (\tau_{\mathit{Color}} \otimes \tau_{\mathit{Year}})^{\circ}$$

where N stands for the second matrix of the previous slide, yielding

	Blue			*Green*			*Red*			ALL		
	1990	1991	ALL	1990	1991	ALL	1990	1991	ALL	1990	1991	ALL
Chevy	87	0	87	0	0	0	5	0	5	92	0	92
Ford	99	7	106	64	0	64	0	8	8	163	15	178
ALL	186	7	193	64	0	64	5	8	13	255	15	270

See how the 36 entries of the original cube have been rearranged in a 3*12 rectangular layout, as dictated by the **dimension** cardinalities.

The **cube** (LA) operator

Definition (Cube)

Let M be a matrix of type

$$\Pi_{j=1}^{n} B_j \xleftarrow{M} \Pi_{i=1}^{m} A_i \tag{10}$$

We define matrix **cube** M, the *cube of* M, as follows

$$\textbf{cube}\ M \quad = \quad (\bigotimes_{j=1}^{n} \tau_{B_j}) \cdot M \cdot (\bigotimes_{i=1}^{m} \tau_{A_i})^{\circ} \tag{11}$$

where $\bigotimes$ is finite Kronecker product.

So **cube** M has type $\Pi_{j=1}^{n}(B_j + 1) \longleftarrow \Pi_{i=1}^{m}(A_i + 1)$.

□

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Properties of data cubing

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Linearity:

$$\textbf{cube}\ (M + N) \quad = \quad \textbf{cube}\ M + \textbf{cube}\ N \tag{12}$$

Proof: Immediate by bilinearity of matrix composition:

$$M \cdot (N + P) \quad = \quad M \cdot N + M \cdot P \tag{13}$$

$$(N + P) \cdot M \quad = \quad N \cdot M + P \cdot M \tag{14}$$

This can be taken advantage of not only in **incremental** data cube construction but also in **parallelizing** data cube generation.

Properties of data cubing

Updatability: by Khatri-Rao product linearity,

$$(M + N) \triangledown P = M \triangledown P + N \triangledown P$$
$$P \triangledown (M + N) = P \triangledown M + P \triangledown N$$

the **cube** operator commutes with the usual CRUDE operations, namely record **updating**. For instance, suppose record

#	Model	Year	Color	Sale
5	*Ford*	1991	*Red*	8

cf

t_{Model}	1	2	3	4	5	6
Chevy	1	1	0	0	0	0
Ford	0	0	1	1	1	1

is updated to

#	Model	Year	Color	Sale
5	*Chevy*	1991	*Red*	8

cf

t'_{Model}	1	2	3	4	5	6
Chevy	1	1	0	0	1	0
Ford	0	0	1	1	0	1

Properties of data cubing

One just has to compute the "delta" projection,

$$\delta_{Model} = t'_{Model} - t_{Model} =$$

	1	2	3	4	5	6
Chevy	0	0	0	0	1	0
Ford	0	0	0	0	-1	0

then the "delta cube",

$$d = (\tau_{Year} \otimes (\tau_{Color} \otimes \tau_{Model})) \cdot v'$$

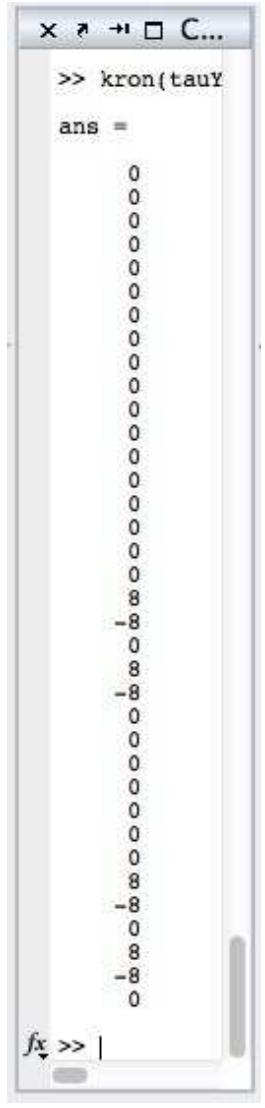
where

$$v' = (t_{Year} \triangledown (t_{Color} \triangledown \delta_{Model})) \cdot (t^{Sale})^{\circ}$$

and finally **add** the "delta cube" to the original cube:

$$c' = c + d.$$

— see the simulation aside (MATLab).



Properties of data cubing

Commutativity: cube commutes with **vectorization**:

Let $X \xleftarrow{M} Y \times C$ *and* $Y \times X \xleftarrow{\mathbf{vec}\, M} C$ *be its* Y*-vectorization. Then*

$$\mathbf{vec}\,(\mathbf{cube}\ M) = \mathbf{cube}\ (\mathbf{vec}\, M) \tag{15}$$

holds. $\square$

Type diagrams:

$$\begin{array}{ccc} Y \times X & \xleftarrow{\mathbf{vec}_Y\, M} & C \\ {\scriptstyle \tau_Y \otimes \tau_M}\downarrow & & \uparrow{\scriptstyle \tau_C^\circ} \\ (Y+1) \times (X+1) & \underset{\mathbf{vec}_{Y+1}\,(\mathbf{cube}\ M)}{\overset{\mathbf{cube}\,(\mathbf{vec}_Y\, M)}{\rightleftarrows}} & C+1 \end{array} \qquad \cong \qquad \begin{array}{ccc} X & \xleftarrow{M} & Y \times C \\ {\scriptstyle \tau_X}\downarrow & & \uparrow{\scriptstyle (\tau_Y \otimes \tau_C)^\circ} \\ X+1 & \xleftarrow{\mathbf{cube}\, M} & (Y+1) \times (C+1) \end{array}$$

Proof in (Oliveira and Macedo, 2017).

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Properties of data cubing

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The following theorem shows that changing the dimensions of a data cube does not change its totals.

Theorem (Free theorem)

Let $B \xleftarrow{M} A$ *be cubed into* $B + 1 \xleftarrow{\textbf{cube } M} A + 1$ *, and* $r : C \to A$ *and* $s : D \to B$ *be arbitrary functions. Then*

$$\textbf{cube}\ (s^{\circ} \cdot M \cdot r) \quad = \quad (s^{\circ} \oplus id) \cdot (\textbf{cube}\ M) \cdot (r \oplus id) \qquad (16)$$

holds, where $M \oplus N = \begin{bmatrix} M & 0 \\ 0 & N \end{bmatrix}$ *is matrix* **direct sum**.

□

The proof given in the same paper resorts to the **free theorem** of **polymorphic** operators due to J. Reynolds and popularized by Wadler (1989) under the heading **Theorems for free!**.

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Cube universality — slicing

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Slicing is a specialized filter for a particular value in a dimension.

Suppose that from our starting cube

$$c : 1 \to (\mathit{Year} + 1) \times ((\mathit{Color} + 1) \times (\mathit{Model} + 1))$$

one is only interested in the data concerning year 1991.

It suffices to regard data values as (categorial) **points**: given $p \in A$, constant function $\underline{p} : 1 \to A$ is said to be a *point* of A, for instance

$$\underline{1991} : 1 \to \mathit{Year} + 1$$

$$\underline{1991} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

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Cube universality — slicing

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Example:

$$1 \xrightarrow{c} (Year+1) \times ((Color+1) \times (Model+1)) \xrightarrow{\underline{1991}^{\circ} \otimes id} 1 \times ((Color+1) \times (Model+1)) = \begin{bmatrix} 0 \\ 7 \\ 7 \\ 0 \\ 0 \\ 0 \\ 0 \\ 8 \\ 8 \\ 0 \\ 15 \\ 15 \end{bmatrix}$$

Cube universality — rolling-up

Gray et al. (1997) say that *going up the levels [of aggregated data] is called rolling-up*.

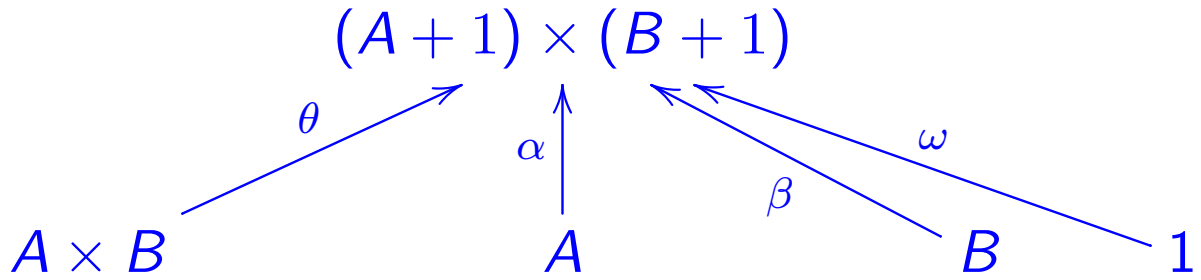
In this sense, a **roll-up** operation over dimensions A, B and C could be the following form of (increasing) summarization:

$$A \times (B \times C)$$
$$A \times B$$
$$A$$
$$1$$

How does this work over a data cube? We take the simpler case of two dimensions A, B as example.

Cube universality — rolling-up

The dimension powerset for A, B is captured by the corresponding matrix **injections** onto the cube target type $(A+1) \times (B+1)$:



where

$$\theta = i_1 \otimes i_1$$
$$\alpha = i_1 \triangledown i_2 \cdot !$$
$$\beta = i_1 \cdot ! \triangledown i_2$$
$$\omega = i_2 \triangledown i_2$$

NB: the injections i_1 and i_2 are such that $[i_1|i_2] = id$, where $[M|N]$ denotes the horizonal gluing of two matrices.

Cube universality — rolling-up

One can build compound injections, for instance

$$\rho : (A + 1) \times (B + 1) \leftarrow A \times B + (A + 1)$$
$$\rho = [\theta | [\alpha | \omega]]$$

Then, for $M : C \to A \times B$:

$$\rho^{\circ} \cdot (\textbf{cube}\ M) = \left[\frac{M}{\left[\frac{fst \cdot M}{! \cdot M} \right]} \right] \cdot \tau_C^{\circ}$$

extracts from **cube** M the corresponding **roll-up**.

The next slides give a concrete example.

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Cube universality — rolling-up

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Let M be the (generalized) data cube

		1990	1991	ALL
	Chevy	87	0	87
Blue	*Ford*	99	7	106
	ALL	186	7	193
	Chevy	0	0	0
Green	*Ford*	64	0	64
	ALL	64	0	64
	Chevy	5	0	5
Red	*Ford*	0	8	8
	ALL	5	8	13
	Chevy	92	0	92
ALL	*Ford*	163	15	178
	ALL	255	15	270

Cube universality — rolling-up

Building the injection matrix $\rho = [\theta|\,[\alpha|\omega]]$ for types $\mathit{Color} \times \mathit{Model} + \mathit{Color} + 1 \to (\mathit{Color} + 1) \times (\mathit{Model} + 1)$ we get the following matrix (already transposed):

		Blue			*Green*			*Red*			ALL		
		Chevy	*Ford*	ALL	*Chevy*	*Ford*	ALL	*Chevy*	*Ford*	ALL	*Chevy*	*Ford*	ALL
Blue	*Chevy*	1	0	0	0	0	0	0	0	0	0	0	0
	Ford	0	1	0	0	0	0	0	0	0	0	0	0
Green	*Chevy*	0	0	0	1	0	0	0	0	0	0	0	0
	Ford	0	0	0	0	1	0	0	0	0	0	0	0
Red	*Chevy*	0	0	0	0	0	0	1	0	0	0	0	0
	Ford	0	0	0	0	0	0	0	1	0	0	0	0
	Blue	0	0	1	0	0	0	0	0	0	0	0	0
	Green	0	0	0	0	0	1	0	0	0	0	0	0
	Red	0	0	0	0	0	0	0	0	1	0	0	0
	ALL	0	0	0	0	0	0	0	0	0	0	0	1

Cube universality — rolling-up

Then

$$\rho^{\circ} \cdot \textbf{cube}\ M =$$

		1990	1991	ALL
Blue	*Chevy*	87	0	87
	Ford	99	7	106
Green	*Chevy*	0	0	0
	Ford	64	0	64
Red	*Chevy*	5	0	5
	Ford	0	8	8
	Blue	186	7	193
	Green	64	0	64
	Red	5	8	13
	ALL	255	15	270

Note how a roll-up is a particular "subset" of a cube.

Matrix ρ° performs the (quantitative) selection of such a subset.

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Summary

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There is a **lot** of SCIENCE in CS that CS people simply **ignore**.

Abadir and Magnus (2005) stress on the need for a **standardized** notation for **linear algebra** in the field of econometrics and **statistics**.

This talk suggests such a notation should be **properly typed**.

Since (Macedo and Oliveira, 2013) the author has invested in **typing** linear algebra in a way that makes it closer to modern **typed** languages.

This extends previous efforts on applying LA to **OLAP** (Macedo and Oliveira, 2015)

Opportunities

A lot to be done!

MAPi is the right **environment** for this kind of cross-breeding:

- 3 **universities**
- many **labs**
- different **cultures**, nice cooperation...

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Annex

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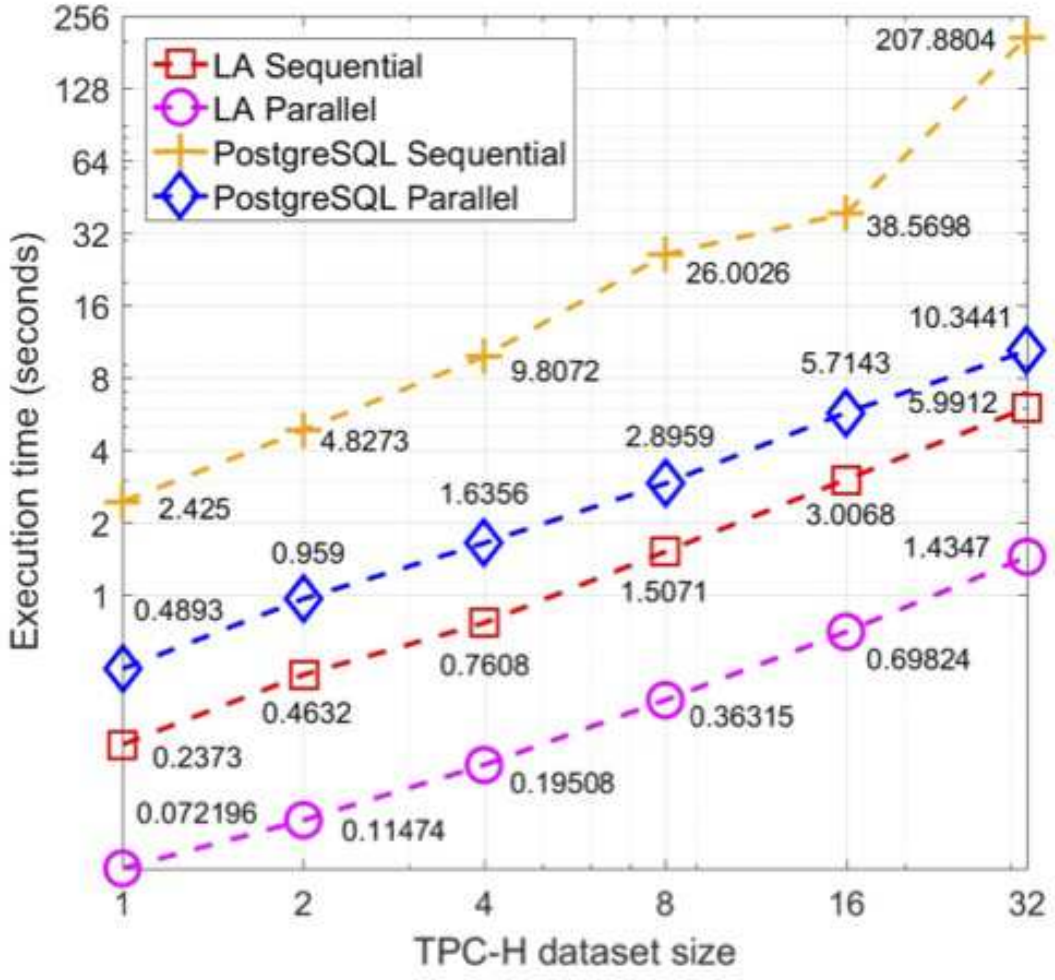
(for those who care mostly about efficiency)

Theory pays off!

Plot taken from a recent MSc report on TPC-H benchmarking LA approach:



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