Algebraic and Coalgebraic Models in Software Development

MAP-I — 2009-10

Summary

This document describes a proposal for the course Fundamentals of Computing *for the 2009-10 MAP-I edition. The proposal is supported by a team from Aveiro University (Dep. of Mathematics) and Minho University (Dep. of Informatics) to be included on the MAP-I* Thematic Seminar on Global Computing*, for 2007-08.*

1 Context and Objectives

An increasing number of computer based systems rely on the cooperation of distributed, heterogeneous components or services organised into open software architectures that, moreover, can survive in loosely-coupled environments and be easily adapted to changing application requirements. Such is the case, for example, of applications designed to take advantage of the increased computational power provided by massively parallel systems or of the whole business of Internet-based software development. In order to develop such systems in a systematic way, the focus of development methods has switched, along the last decade, from functional to architectural issues: both data and processes are encapsulated into software units which are connected into large systems resorting to a number of techniques intended to support reusability and modifiability. This encapsulation principle is essential to both the *object-oriented* and the more recent *component-based* software engineering paradigms.

This entails the need for semantic techniques able to cope either with *date structuring* and *prescription of functionality*, as well as with specification and analysis of (externally observable) *behaviour*.

If on data-intensive applications the main element to tackle is the *structure* of information and its transformations, in dynamic, reactive computing the focus is placed on system's behaviours and their interactions. Quoting Robin Milner, in his Turing Award Lecture, computing science has become a *structural theory of interaction*: *Thus software, from being a prescription for how to do something — in Turing's terms a "list of instructions" — becomes much more akin to a description of behaviour, not only programmed on a computer, but occurring by hap or design inside or outside it.*

Both *initial* algebras and *final* coalgebras provide abstract descriptions of a variety of phenomena in programming, in particular of *data* and *behavioural* structures, respectively. As universal properties, they both entail definitional and proof principles, *i.e.*, a basis for the development of program calculi directly based on (actually driven by) type specifications. Moreover, such properties can be turned into programming *combinators* and used, not only to calculate programs, but also to program with. In functional programming the role of such universals has been fundamental to a whole discipline of algorithm derivation and transformation. On the oher hand, *coalgebraic modelling* of dynamical systems and reasoning by *coinduction* has recently emerged as active area of research.

This course explores the role of such algebraic and coalgebraic structures, and corresponding logics, in program development. As expected, *initial* algebras turn out to be *inductive data types*, *i.e.*, abstract descriptions of data structures. Dually, *final* coalgebras entail a notion of *coinductive*, *behaviour types*, representing the dynamics of systems. Therefore, the course will cover the core ideas, techniques and results in

- Algebraic specification, induction and equational logic
- Coalgebraic specification, coinduction and modal logic for coalgebras

In both cases exposition will resort to suitable tool support (namely, OBJ, BOBJ and Circus).

To provide a common background to formulate and discuss the topics above, the course will also include a brief

- Introduction to category theory

A particular emphasis will be placed on *observational* semantics. As a matter of fact, there are several phenomena in computing which are hardly definable (or even simply not definable) in terms of a complete set of constructors and in an essentially finitary process. Such is the case of, for example, processes, transition systems, objects, stream-like structures used in lazy programming languages, 'infinite' or non well-founded objects arising in semantics. Such 'systems' are inherently dynamic, do possess an observable behaviour, but their internal configurations remain hidden and have therefore to be identified if not distinguishable by observation. Therefore, the study of behavioural satisfaction refinement of algebraic specifications, and coalgebraic methods, will be particularly targeted in the course.

The course will build a *roadmap* to the broad area of algebraic and coalgebraic methods in software development, not only by providing an introductory survey, but also by exposing students to cutting-edge research topics and open problems, eventually leading to the formulation of a few specific research plans.

2 Learning outcomes

- Familiarity with the main topics, research questions and scientific challenges in the covered area (algebraic and coalgebraic methods);
- Ability to apply them to building and reasoning about, abstract models for software, its functionality, behaviour and composition.
- Ability to extract information from scientific papers in the area.
- Enhanced technical writing and presentation skills.

3 Pre-requisites

The course is almost self-contained, assuming only familiarity with elementary discrete mathematics at undergraduate level. Some previous experience on semantics of programming languages will help.

4 Format

Tutorial module, supported with demos and experimental lab work.

5 Grading

Assessment on base of an individual report on a research paper and a set of written exercises.

6 Course Contents

Plan

1. Introduction to category theory for computer science
2. Algebras and algebraic specification
   (a) Signatures, models
   (b) Equational logic
   (c) Signature morphisms
   (d) Refinements
   (e) Introduction to the theory of institutions
   (f) Behavioural specifications
3. Coalgebras and coalgebraic specification
   (a) Coalgebras
   (b) Bisimulation
   (c) Coinduction, final coalgebras
   (d) Logics for coalgebras
   (e) Applications

Textbooks and Reading Material

On category theory : [3, 24, 14, 2]

On algebraic specification : [9, 11, 10, 8, 23]

On coalgebraic modelling and coinduction : [22, 12, 13, 1]

7 Team

Luis Soares Barbosa is Associate Professor, with tenure, at the Department of Informatics of Minho University, and a researcher at CCTC (area of *Theory and Formal Methods*). His research interests are related to program semantics and calculi applied to systems understanding and rigorous software construction. A particular application area concerns the development of formal models and calculi for software components, services and architectures. On this topic he has published over the past 4 years more than 15 papers in several journals and conferences. He has supervised 2 PhD thesis (1 in the area of the current proposal) and is currently supervising 5 PhD projects (2 in the area of the current proposal). *Selected relevant publications* on coalgebraic modelling and coinductive reasoning: [4, 5, 19, 21, 6, 20, 7].

Manuel António Martins is Assistant Professor at the Department of Mathematics of Aveiro University, and a researcher at Matemática & Aplicações centre. His research interests are related to Abstract Algebraic Logic (AAL) and Algebraic Specification of abstract data types; namely on the application of tools and results of AAL to the specification and verification of software systems. On this topic he has published 4 papers in international journals. He has supervised 2 MSc thesis (1 in the area of the current proposal) and is currently supervising 1 PhD project in the area of the current proposal. *Selected relevant publications* on specification and verification of software systems: [15, 18, 16, 17].

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